Towards a Tractable Exact Schedulability Test for Real-Time Scheduling (Part 1)

Artem Burmyakov

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About Myself

Education	
2004, 2007	BSc and MSc in Applied Math, Moscow Engineering Physics Institute
2016	PhD in Computer Science, CISTER & FEUP

Relevant work experience	
Since 2019	Post-doctoral researcher / Assistant professor, Innopolis University, Russia
2016 - 2017	Post-doctoral researcher, Seoul National University, South Korea
2006-2010	Software developer, CERN, Switzerland

Delivering courses
Computer Architecture (core course)
Computer Networks (core course)
Consensus Theory and Concurrent Programming (elective course)
Real-Time Scheduling Theory (elective course)

Research Interests
Real-time multiprocessor scheduling
Optimization of computationally intensive algorithms
Concurrent programming, Static analysis of multithreaded programs

System Model:
Real-Time Sporadic Task

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$$\tau_i = (C_i, D_i, P_i)$$

[Liu and Layland 1973]

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Execution time

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Execution time

Relative deadline

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Execution time

Relative deadline

Minimum time between job releases (or simply period)

[Liu and Layland 1973]

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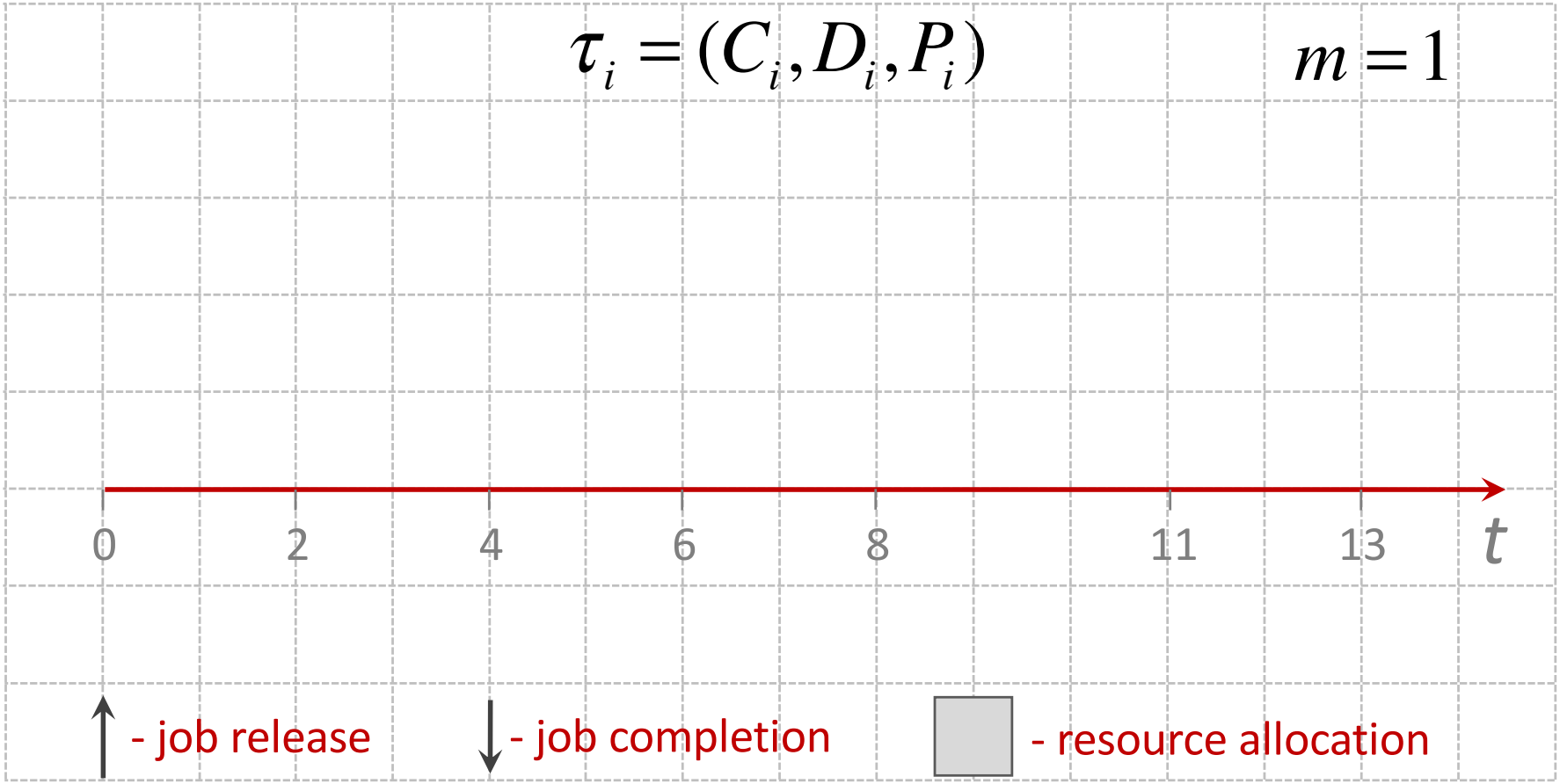

[Liu and Layland 1973]

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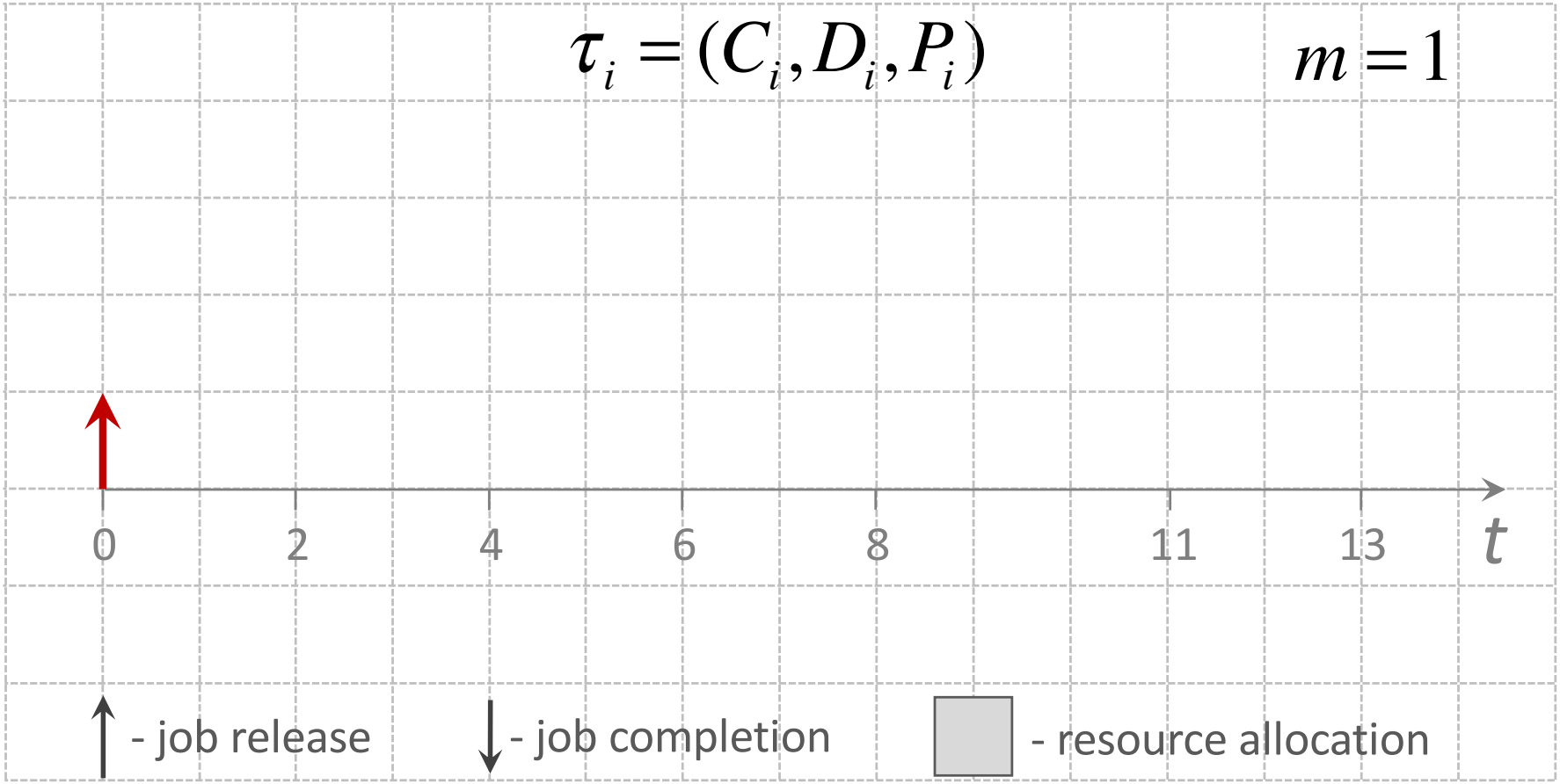

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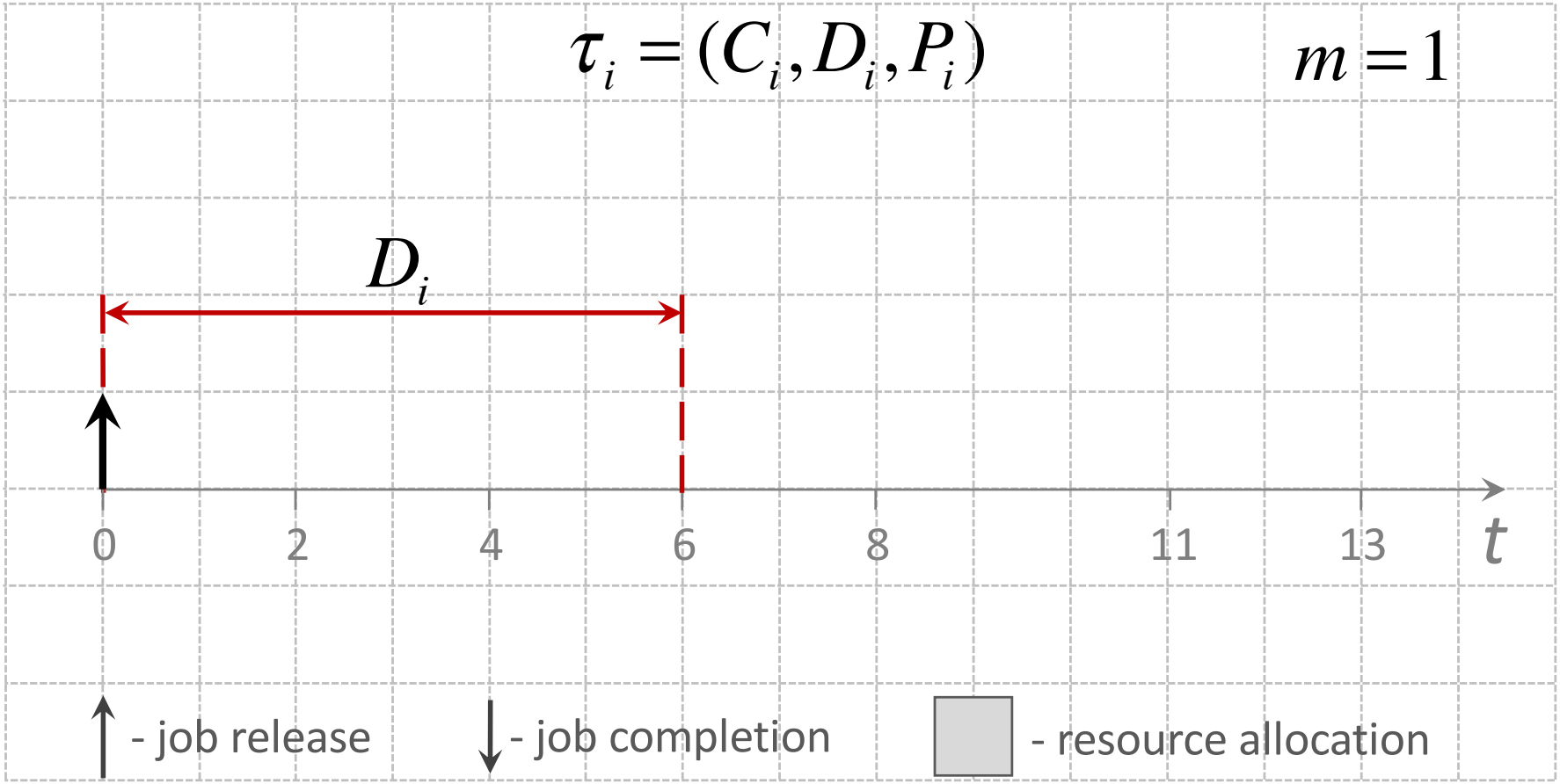

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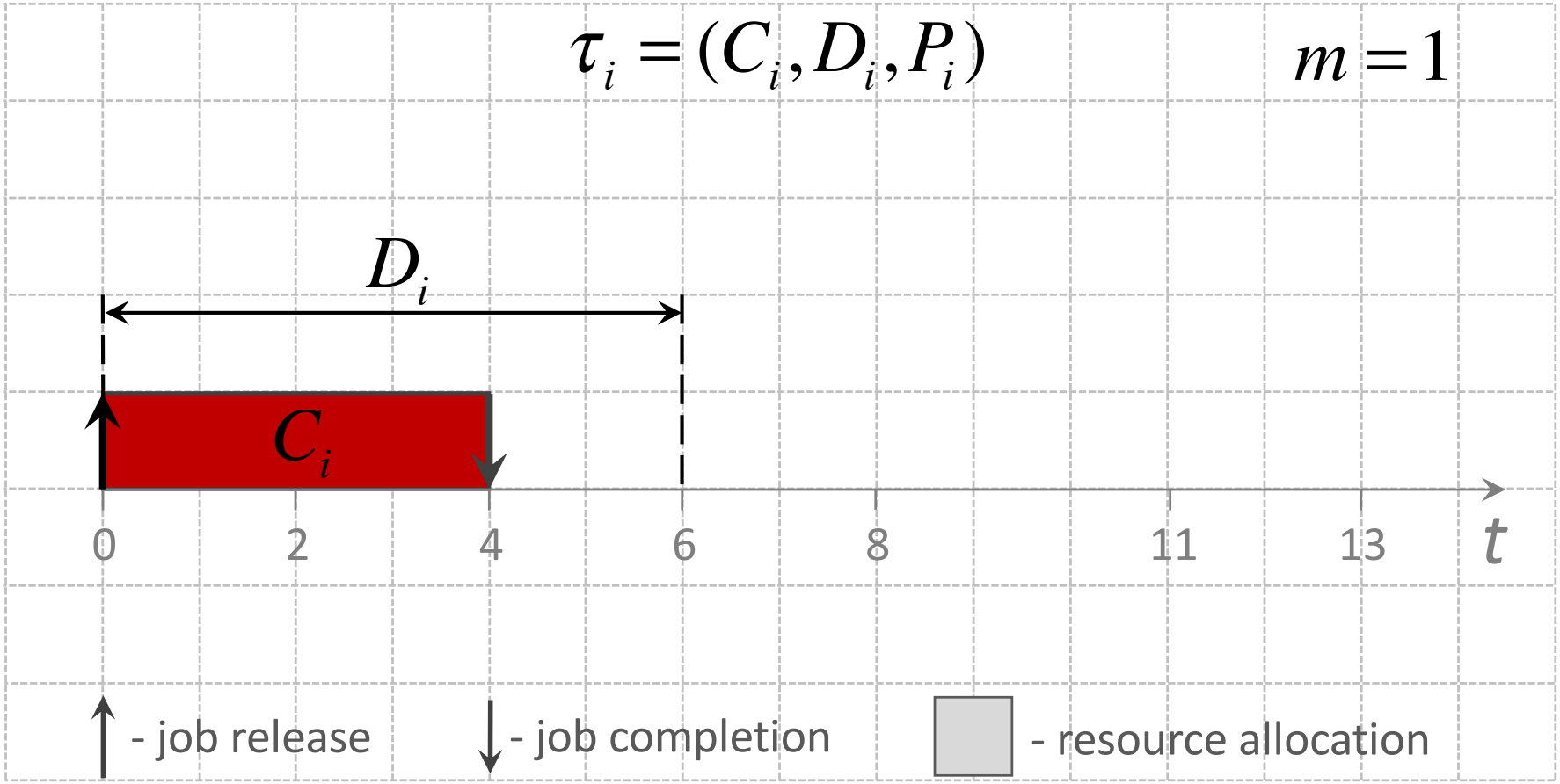

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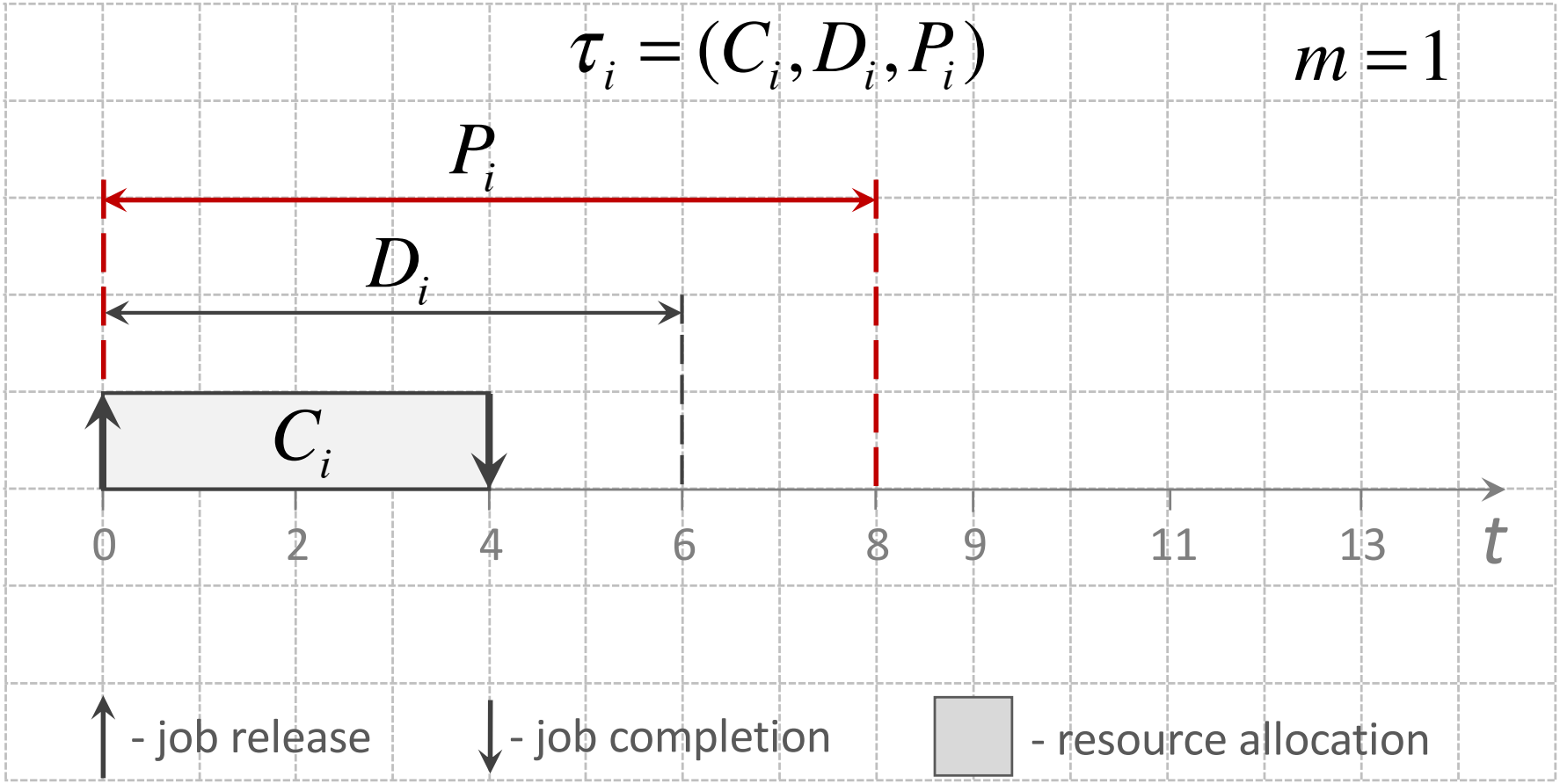

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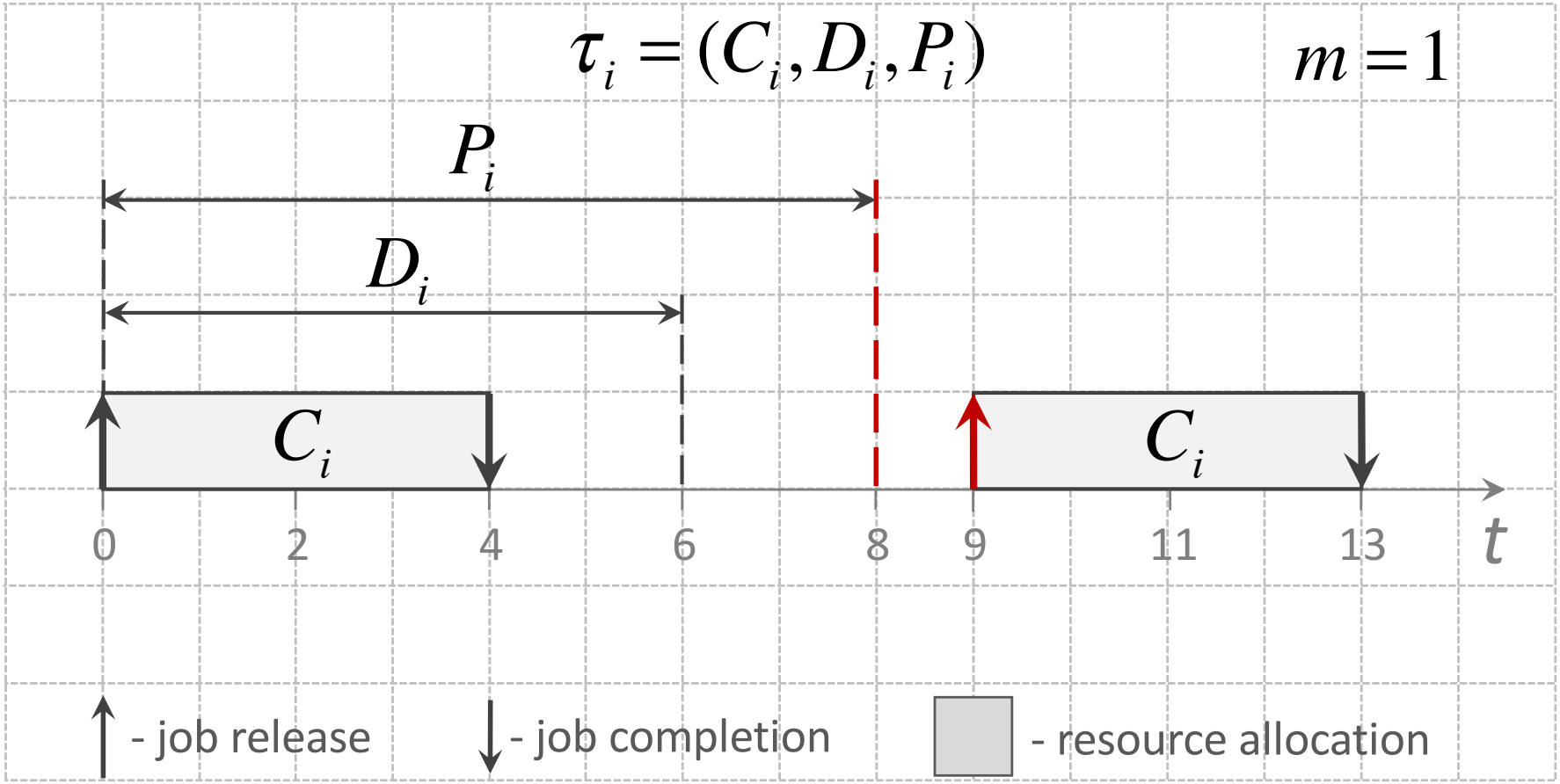
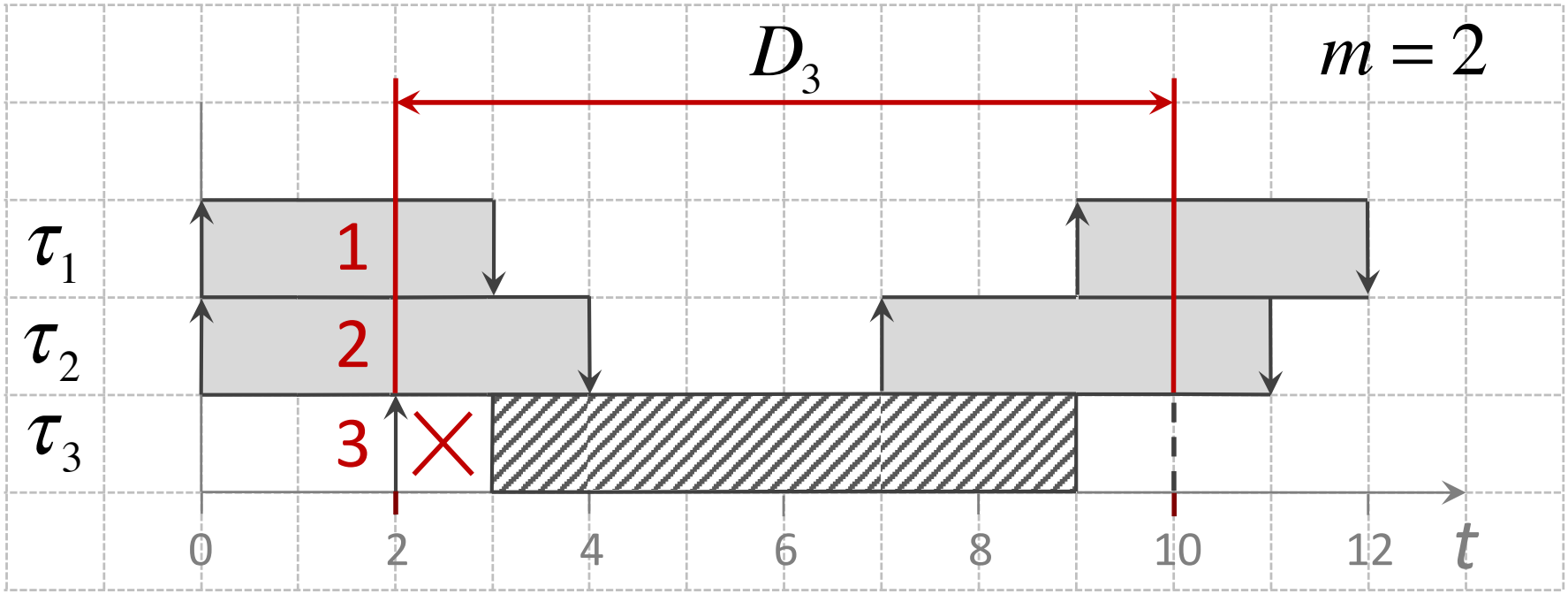


[Liu and Layland 1973]

A Sample Global Fixed Priority (GFP) Schedule

Priorities assumption: Task with a smaller index has a higher priority



- resource available to τ_3

- deadline of τ_3

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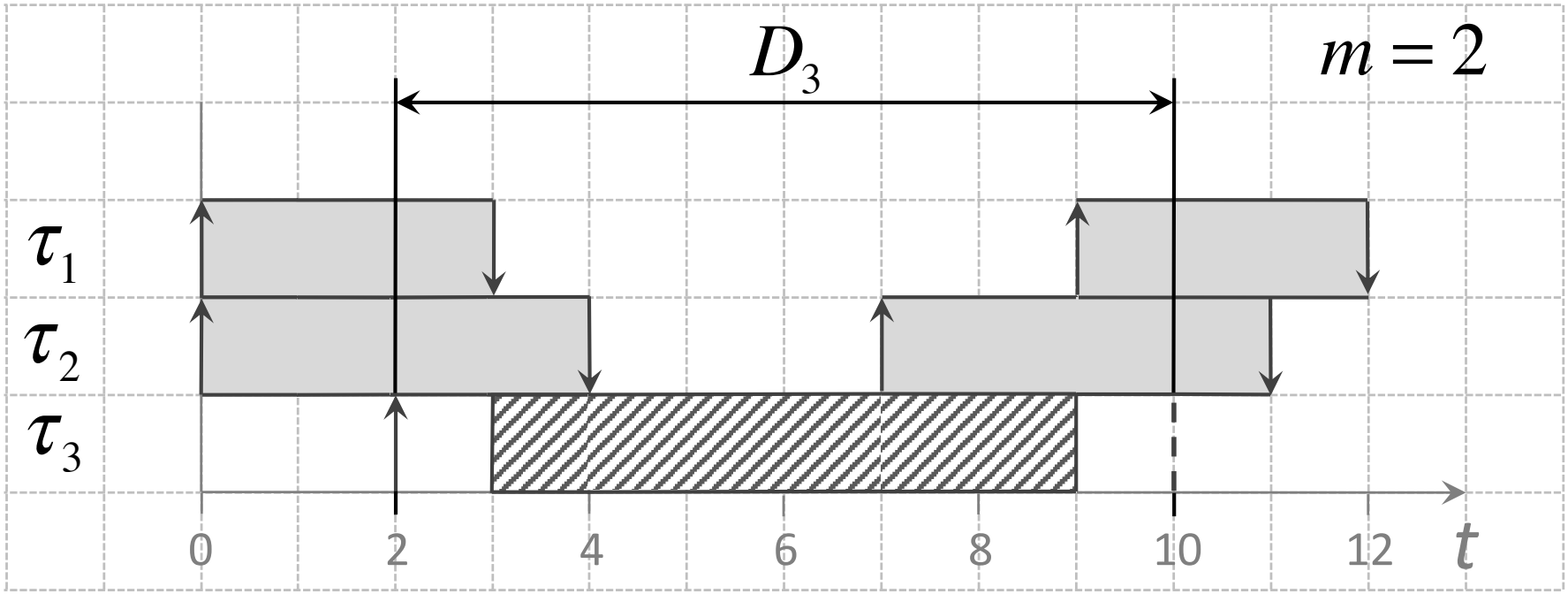

Question: Is there such a scenario, that the task τ_3 misses a deadline?

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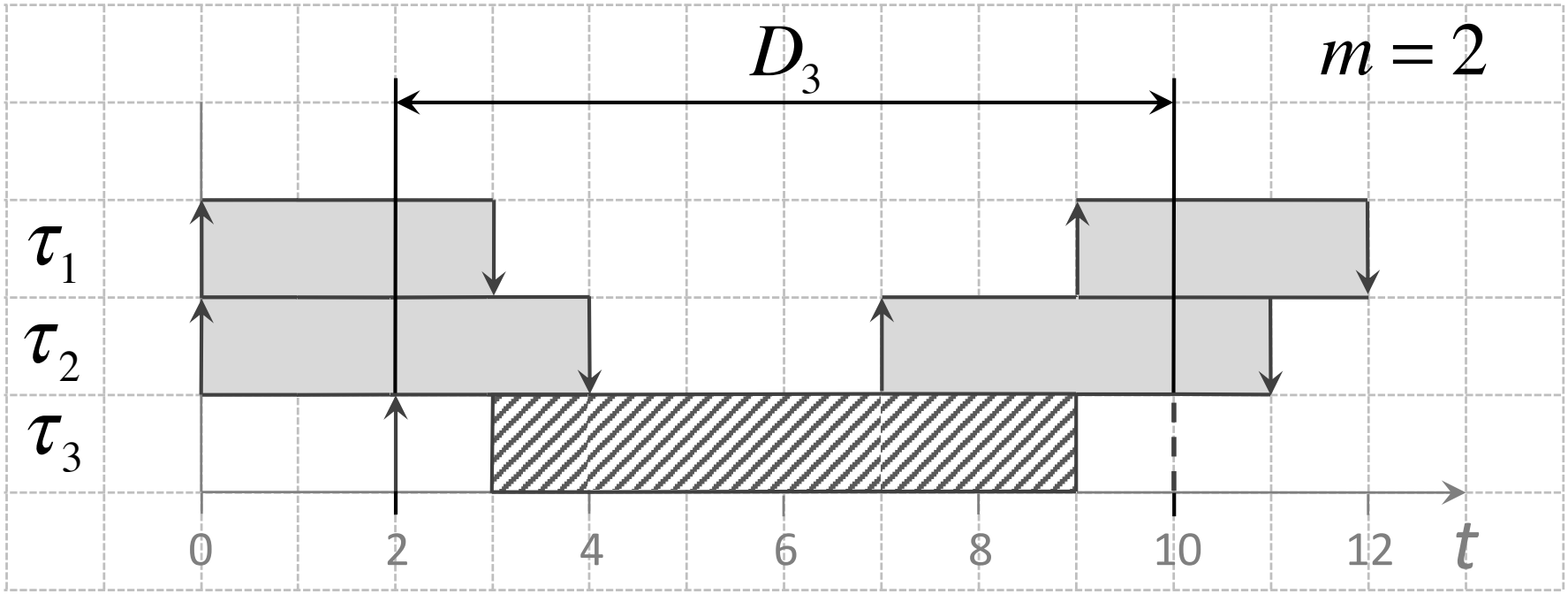

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Possible solutions:

- **Exact (NP-hard)**

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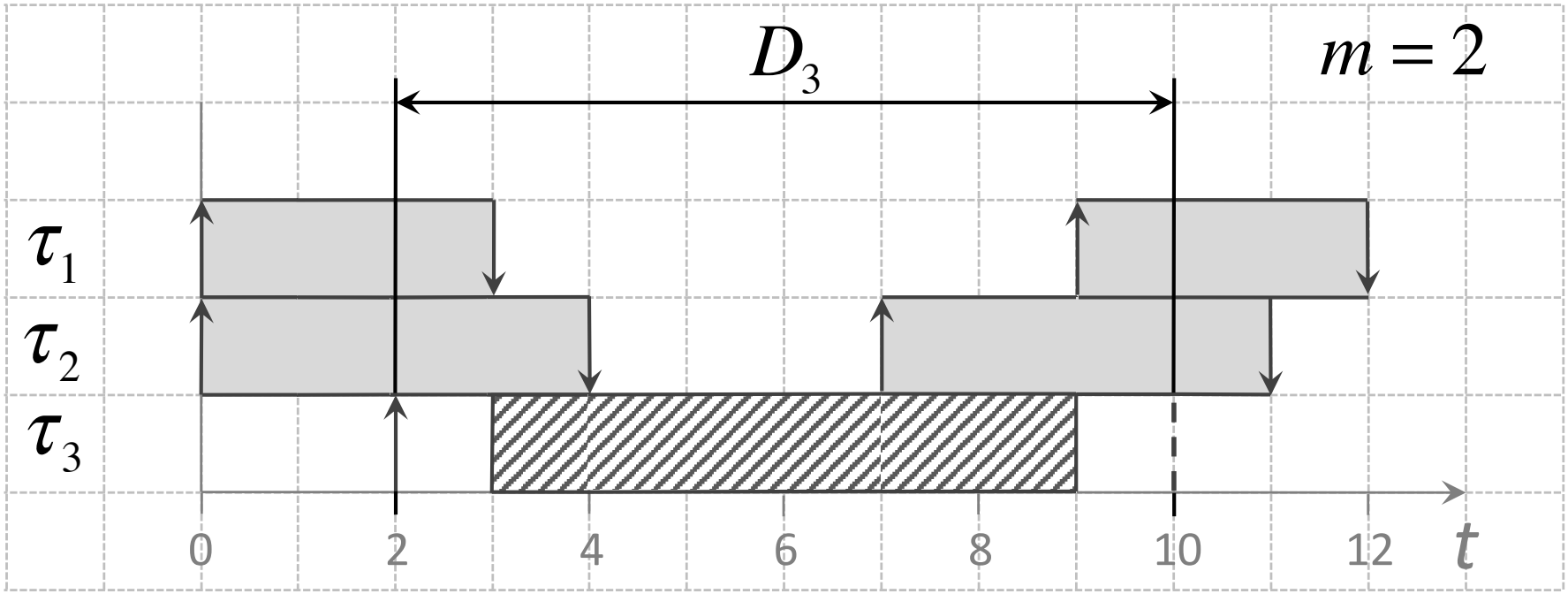


Question: Is there such a scenario, that the task τ_3 misses a deadline?

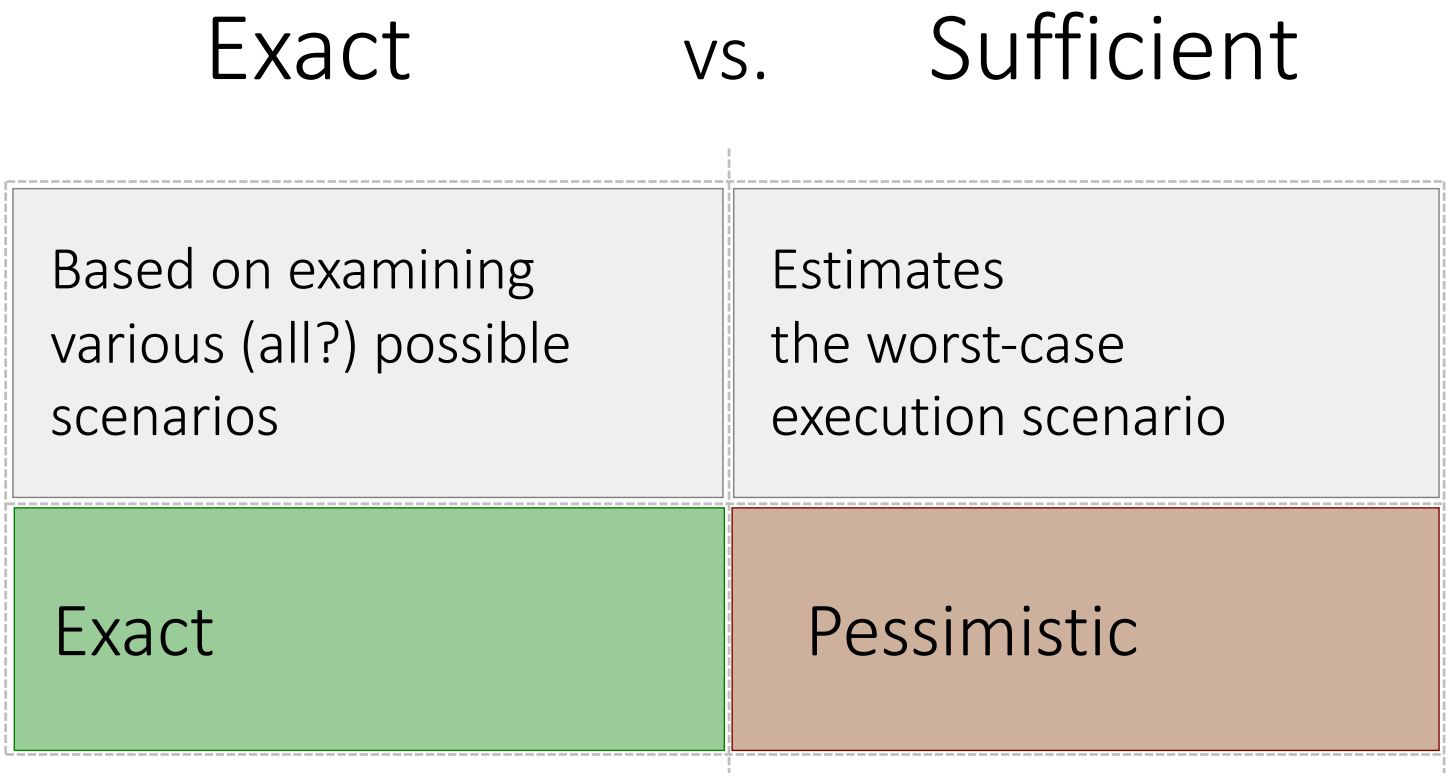
Possible solutions:

- **Exact (NP-hard);**
- **Suboptimal (pessimistic)**

Schedulability Test for Multiprocessors: NP-hard Problem

Exact	vs.	Sufficient
Based on examining various (all?) possible scenarios		Estimates the worst-case execution scenario

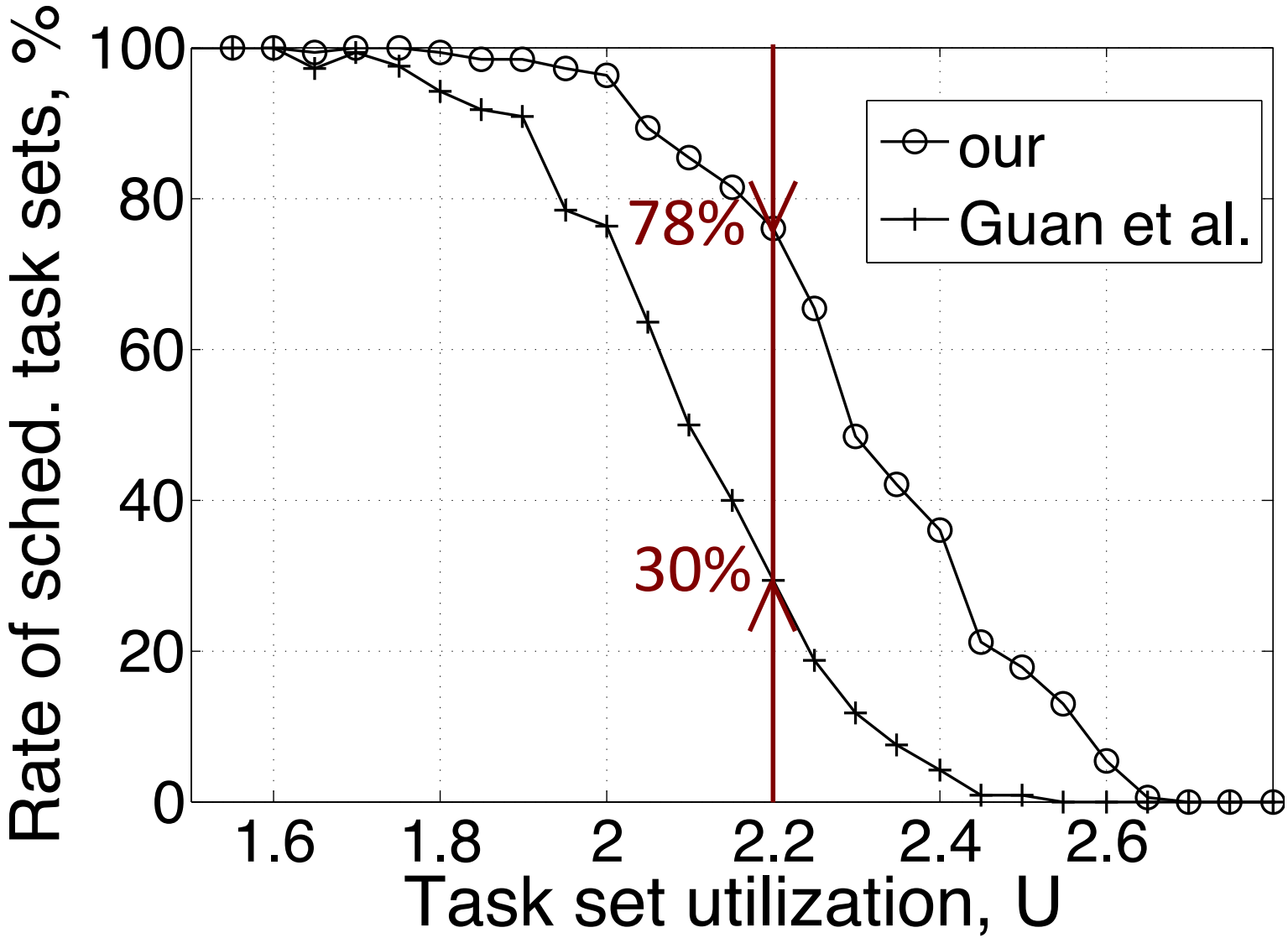
Schedulability Test for Multiprocessors: NP-hard Problem



Schedulability Test for Multiprocessors: NP-hard Problem

Exact	vs.	Sufficient
Based on examining various (all?) possible scenarios		Estimates the worst-case execution scenario
Exact		Pessimistic
Exponential runtime complexity		Lower (<= pseudopolynomial) runtime complexity

Exact vs. Sufficient Schedulability Tests



$m = 3$

$n = 7$

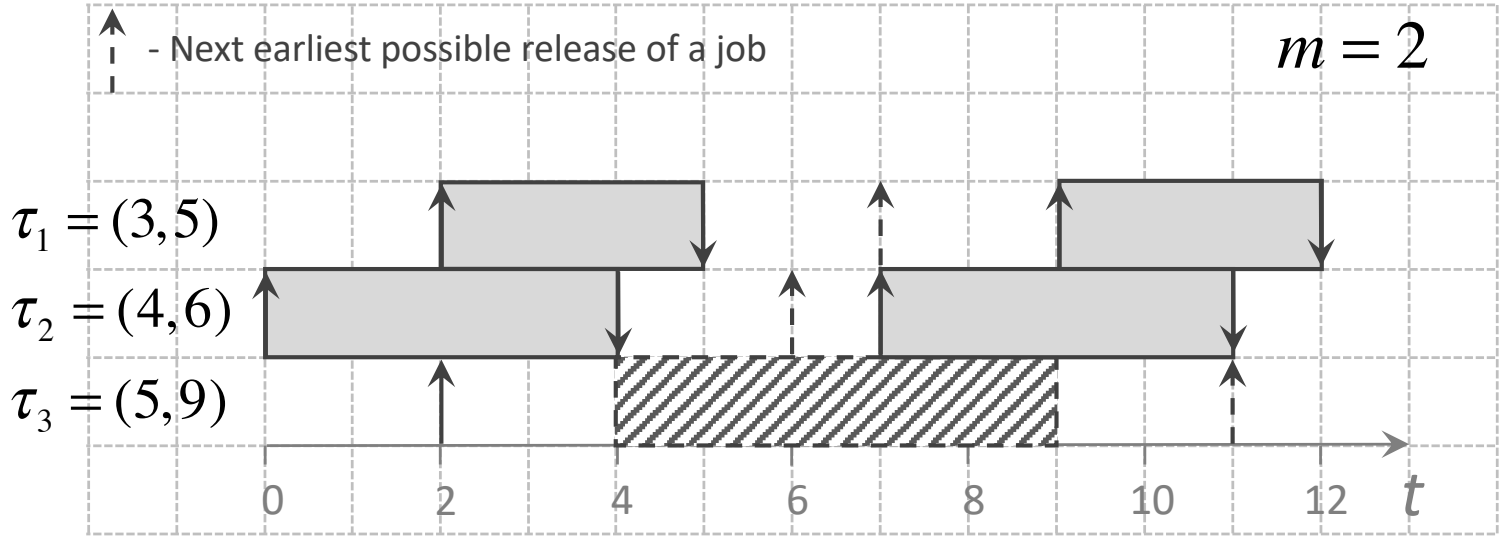
$U_{\max} = 0.6$

$P_{\max} / P_{\min} = 4$

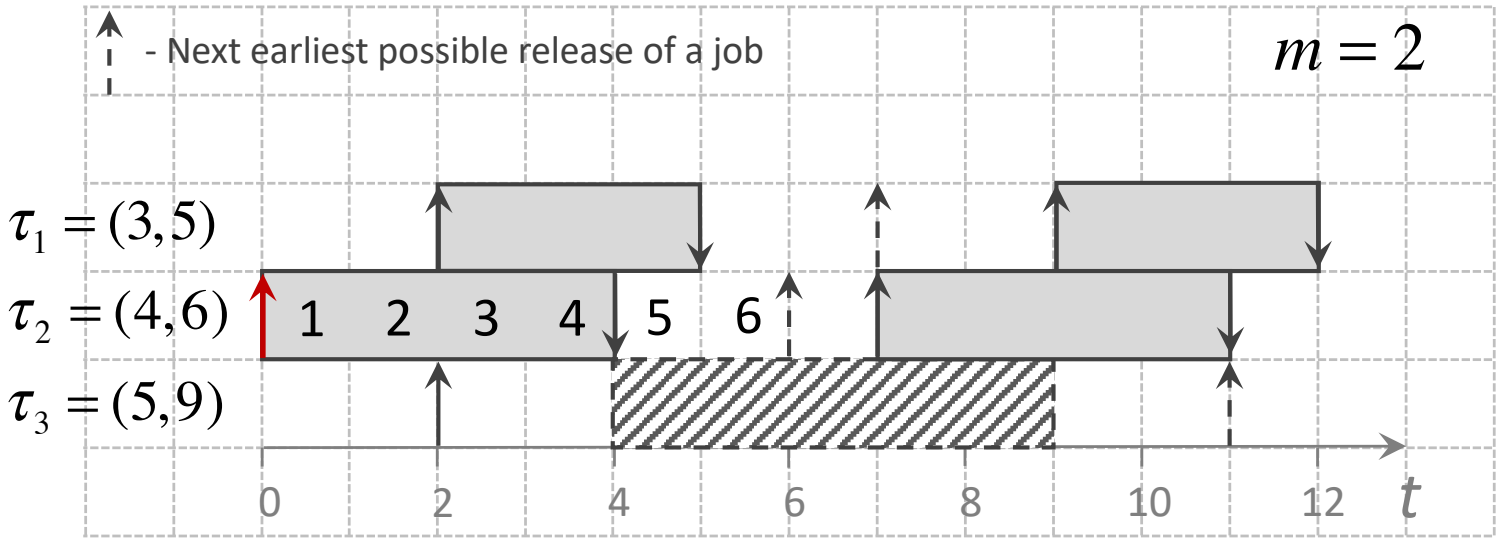
$P_{\min} \in [3,10]$

State Transition Graph

Consider the following GFP schedule:



State Transition Graph for a Schedule



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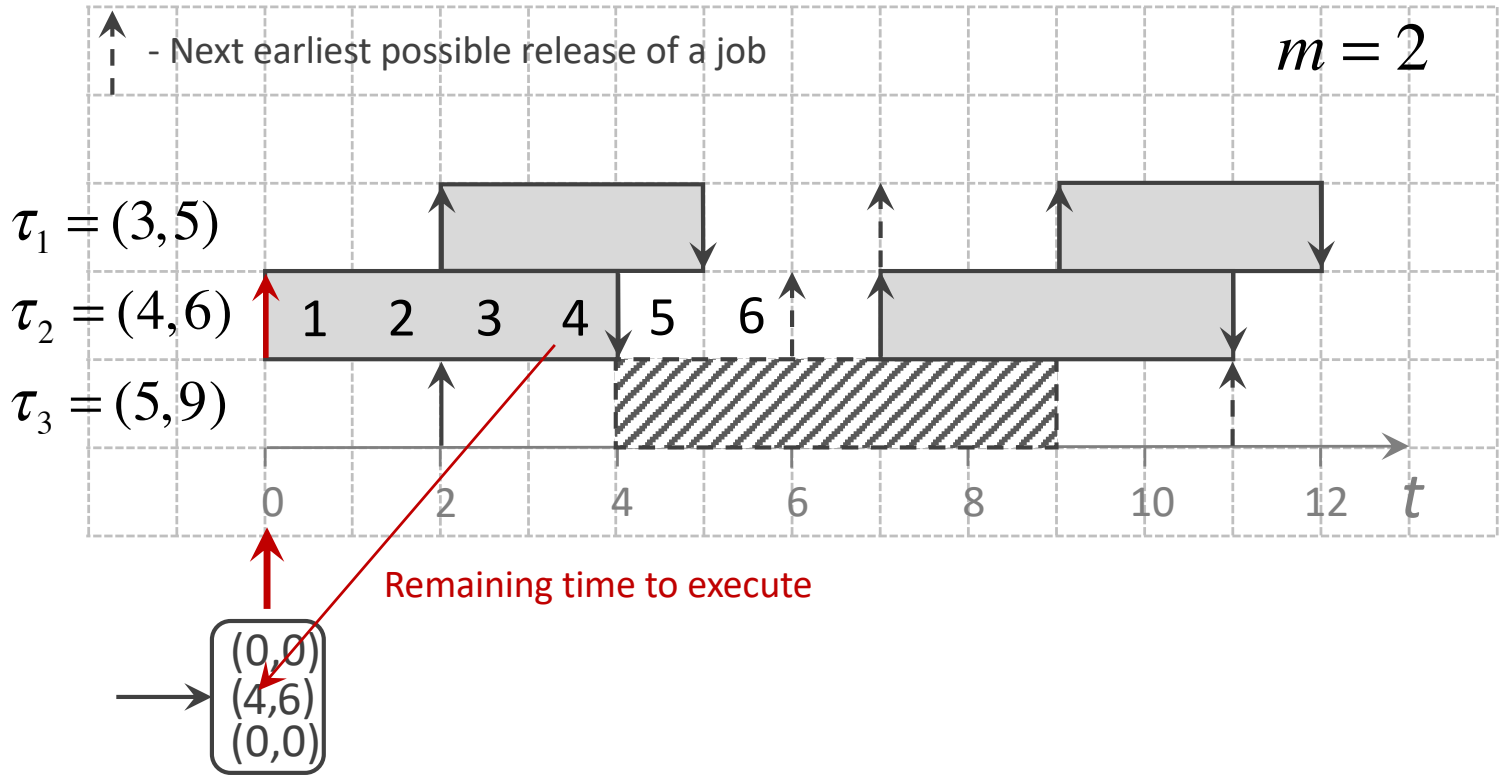

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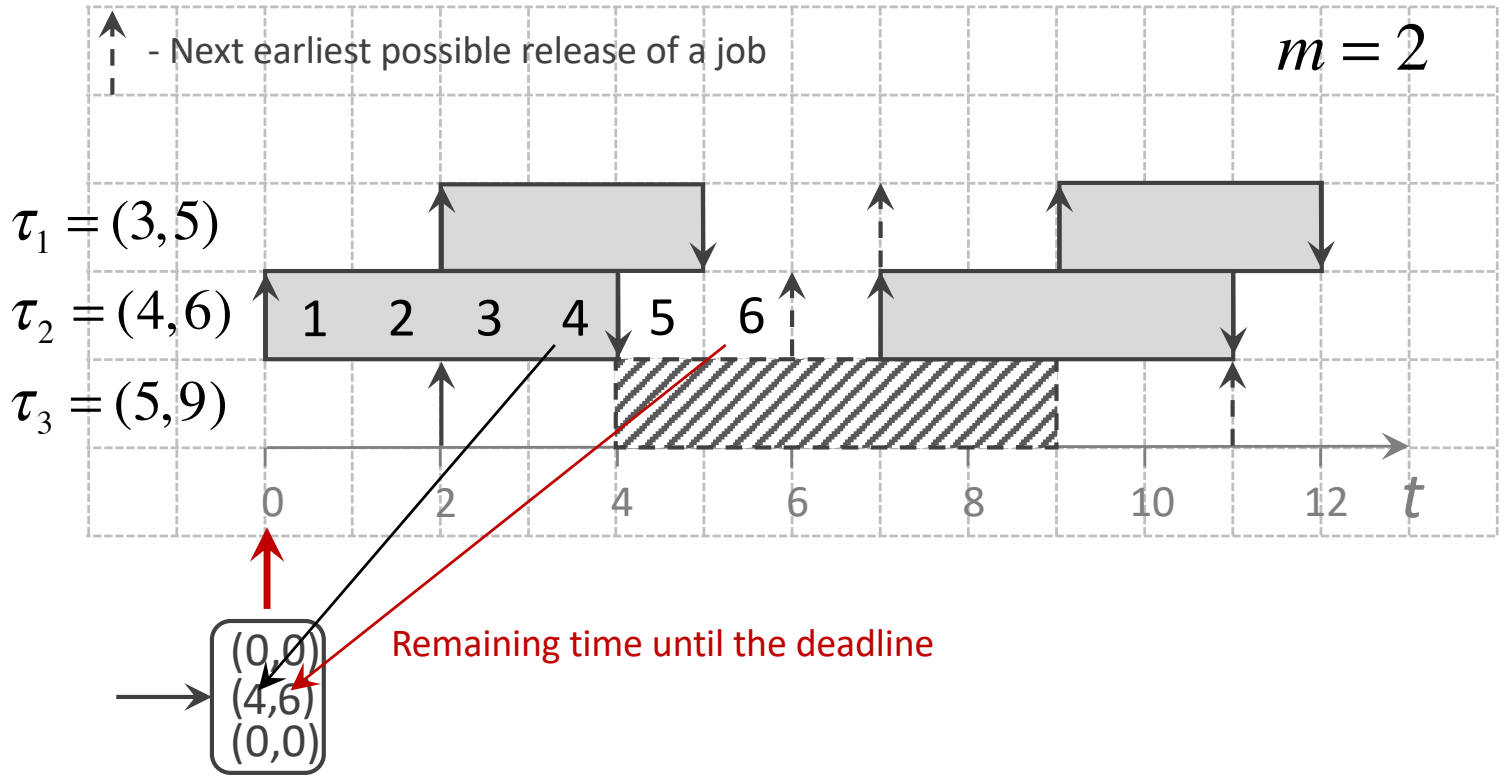

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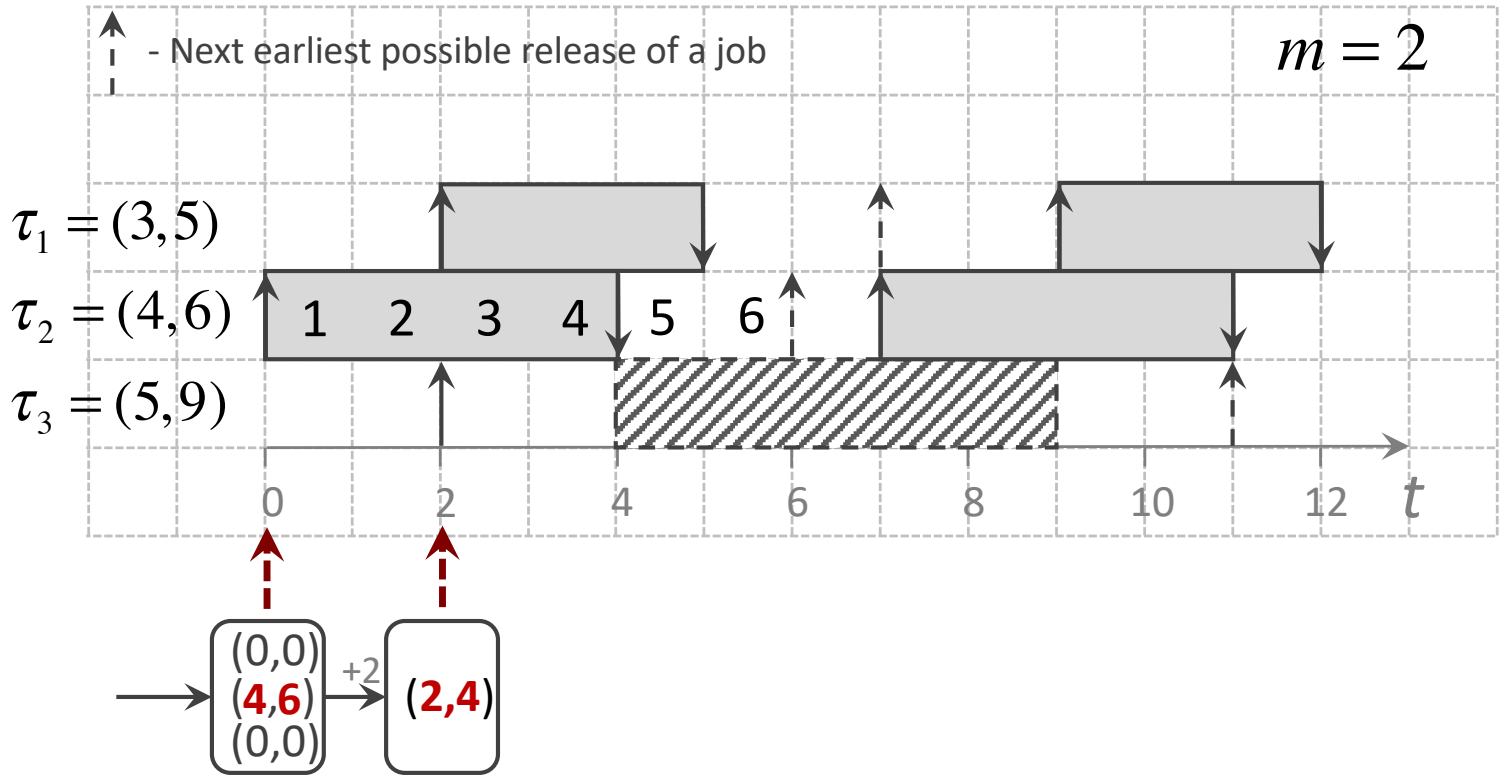

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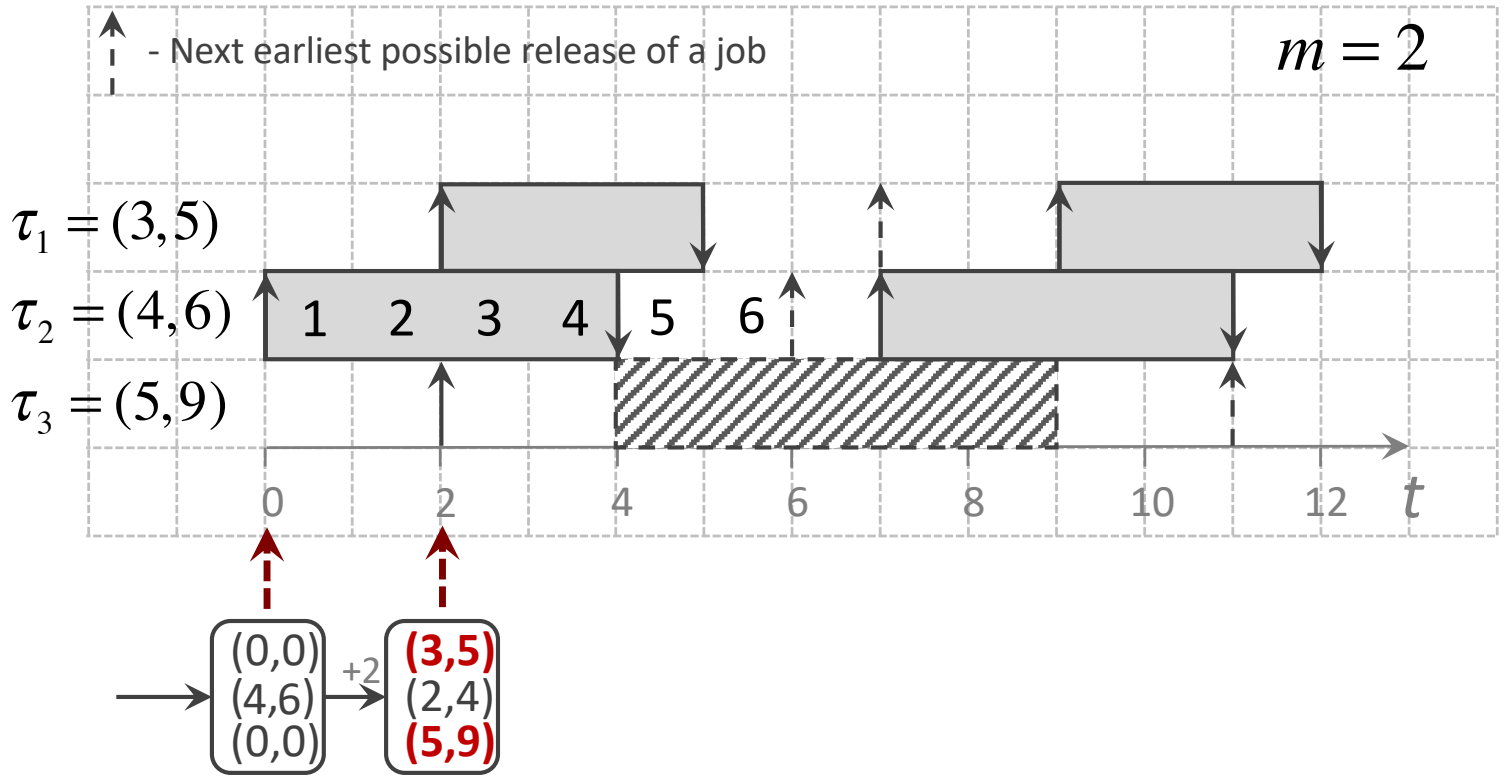

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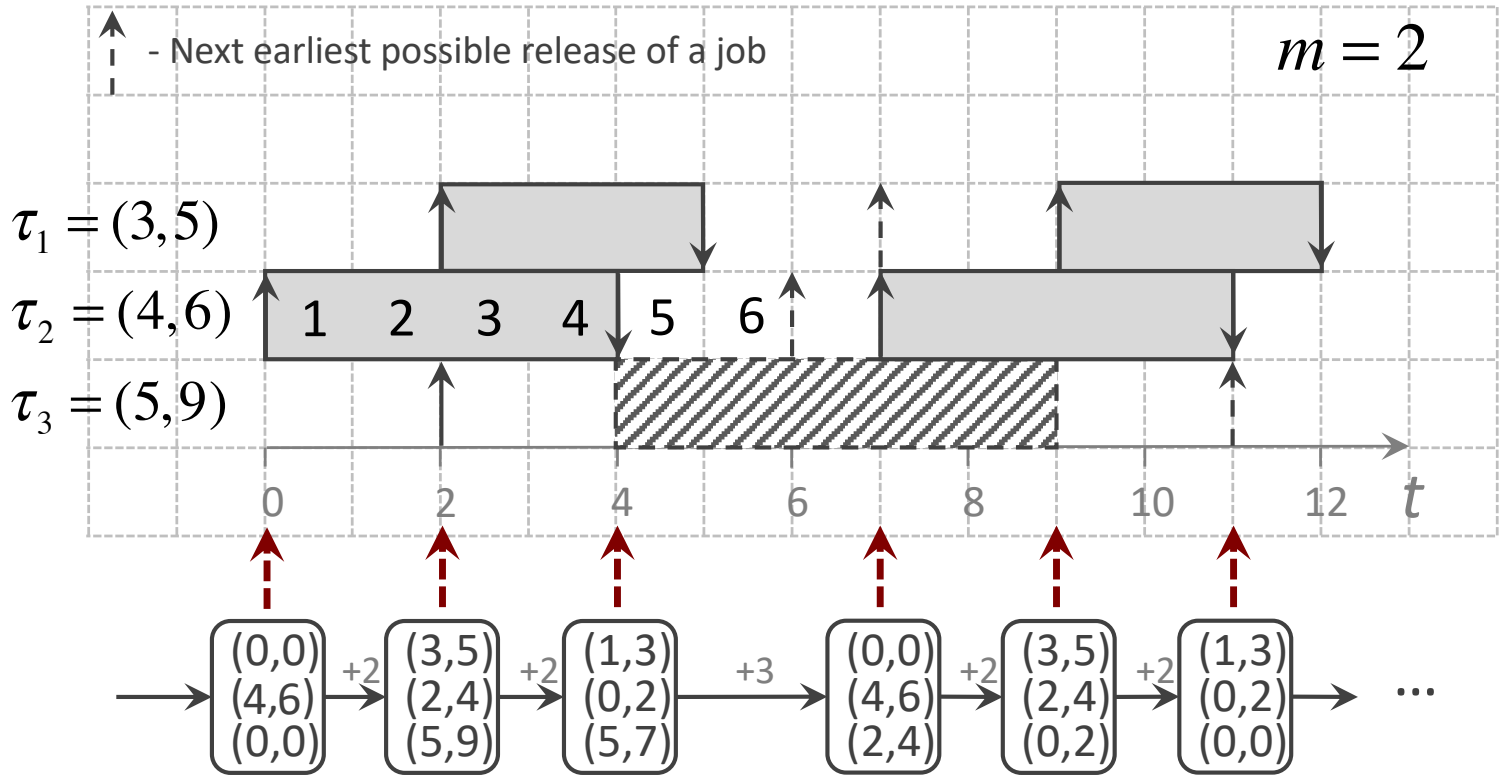

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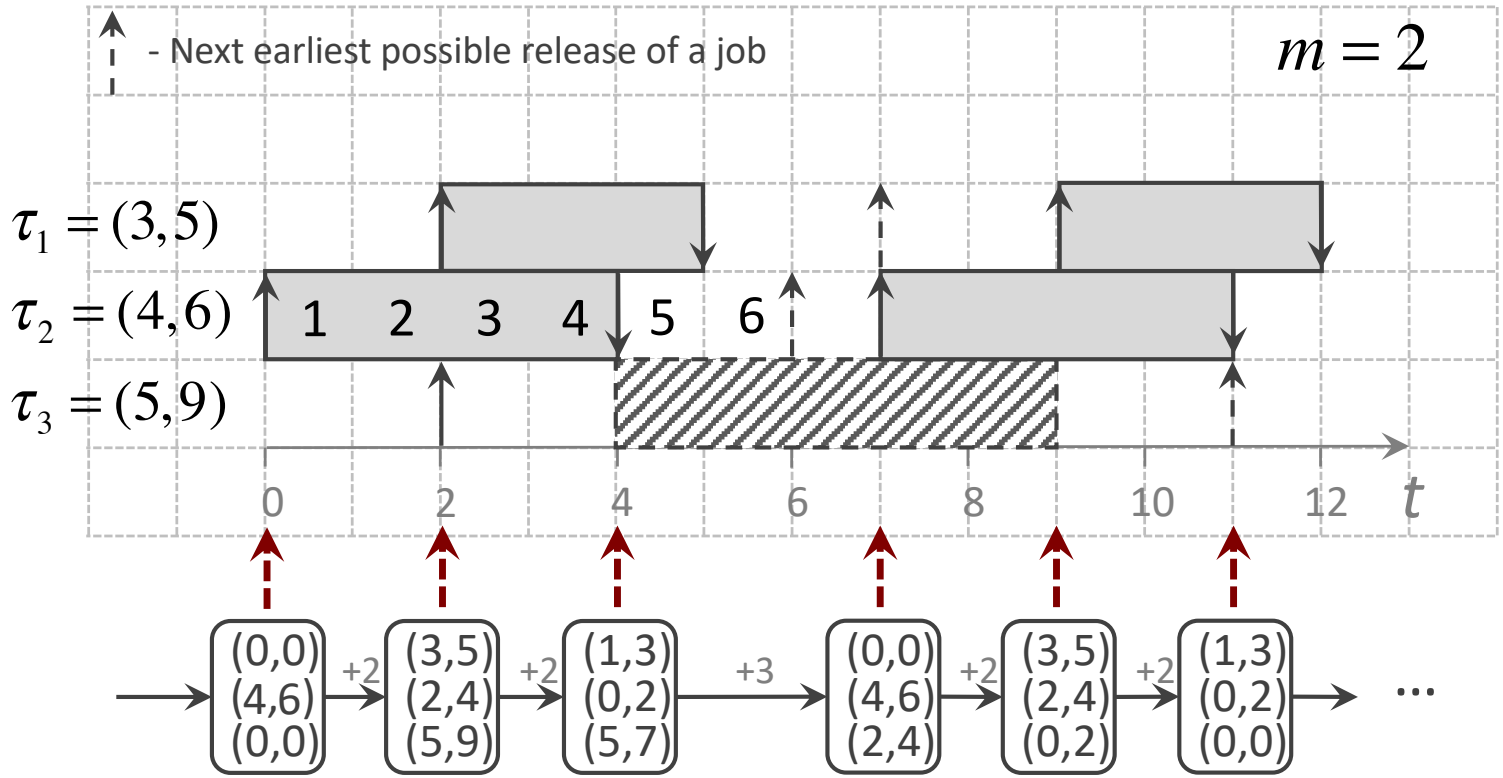
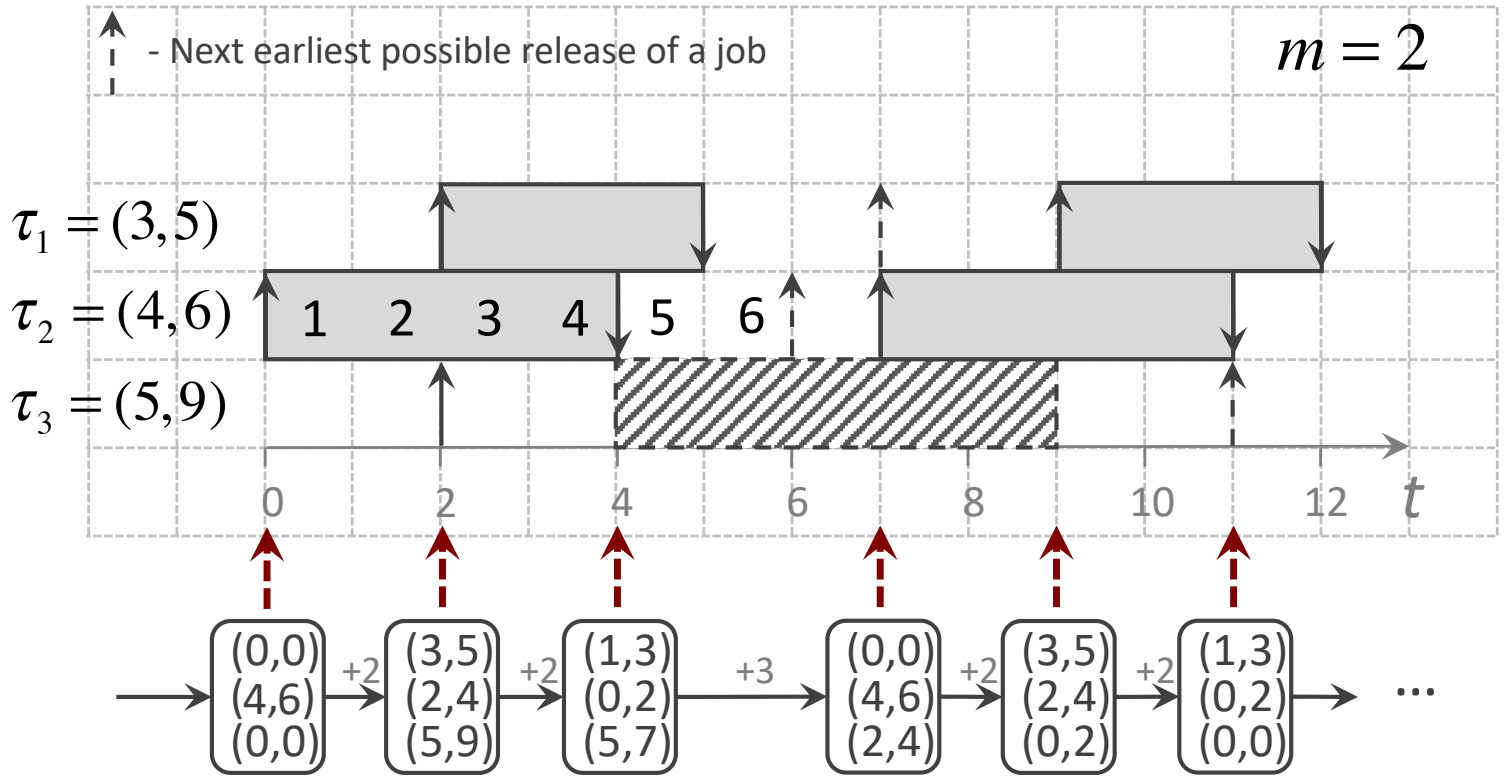


Note: The release scenario above is just one out of many legal release scenarios

State Transition Graph



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State transition graph presented on the next slides aims at modeling all possible schedules

State Transition Graph Example

$\tau_1 = (2,5)$

$\tau_2 = (3,7)$

$m = 1$

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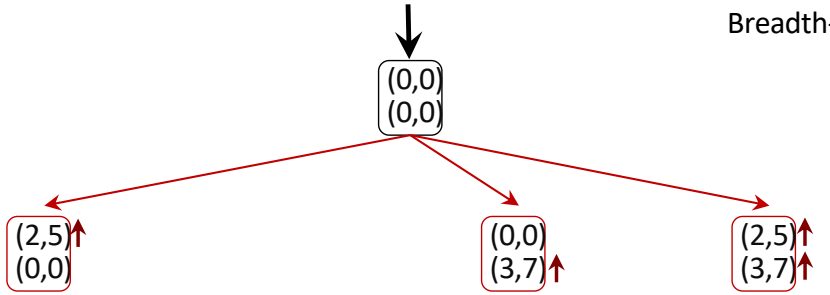

Breadth-First Traversal (BFT)

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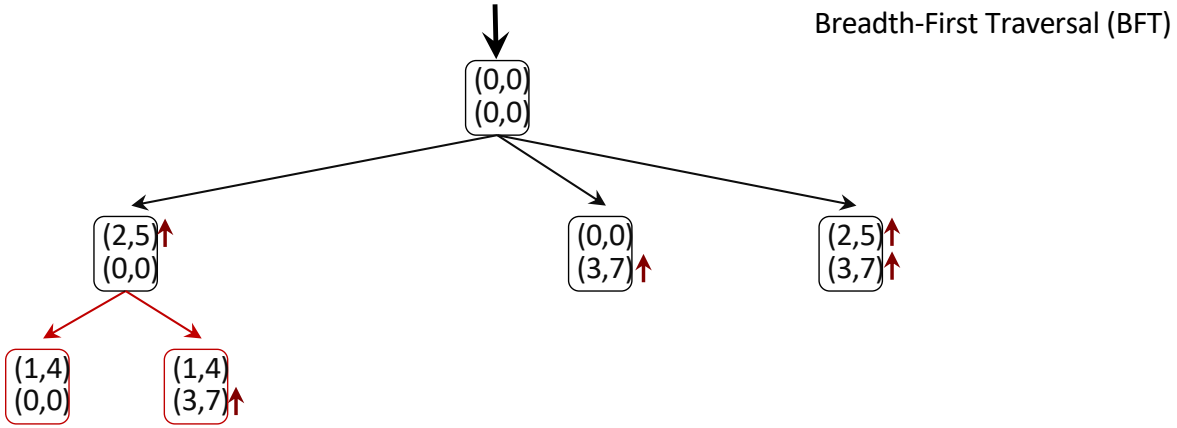

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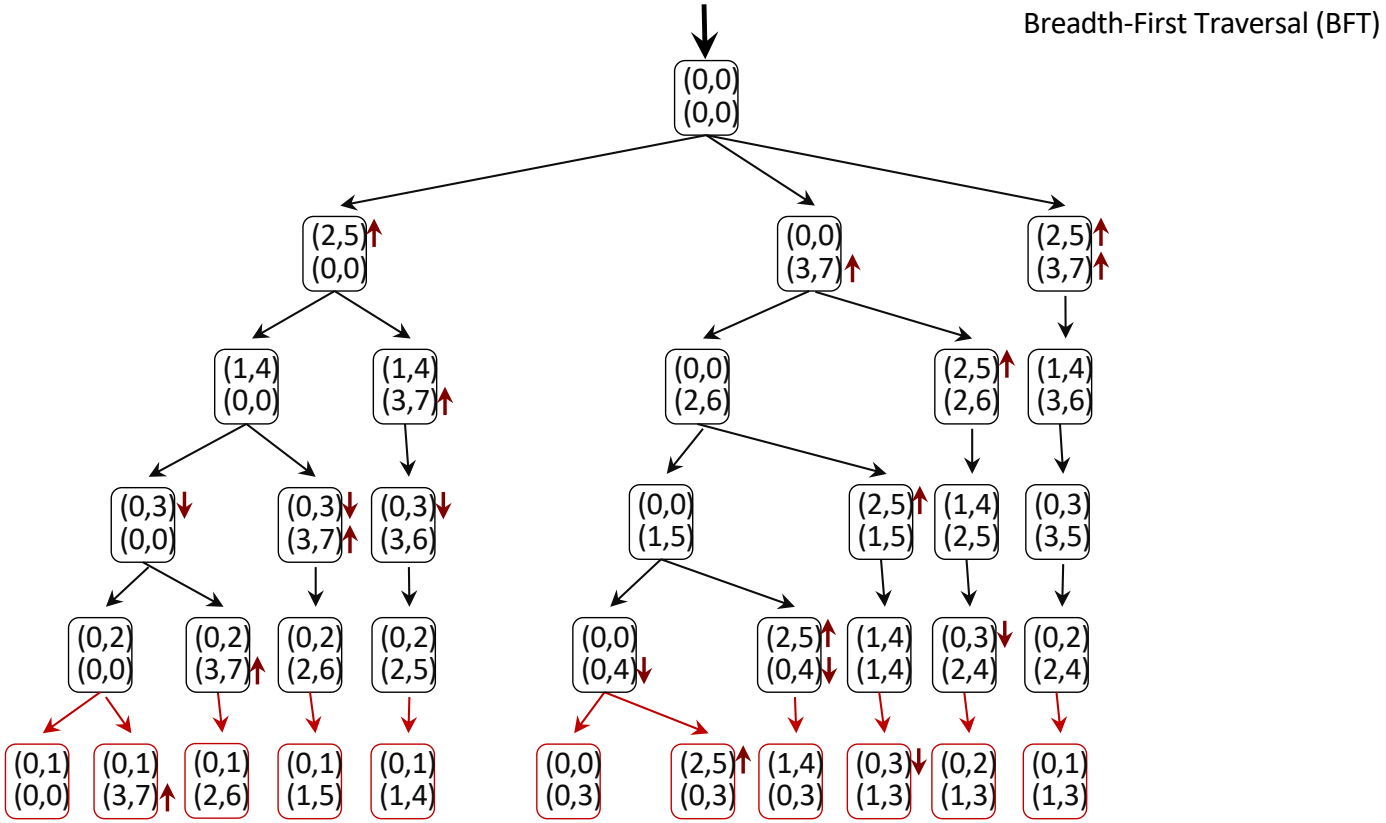

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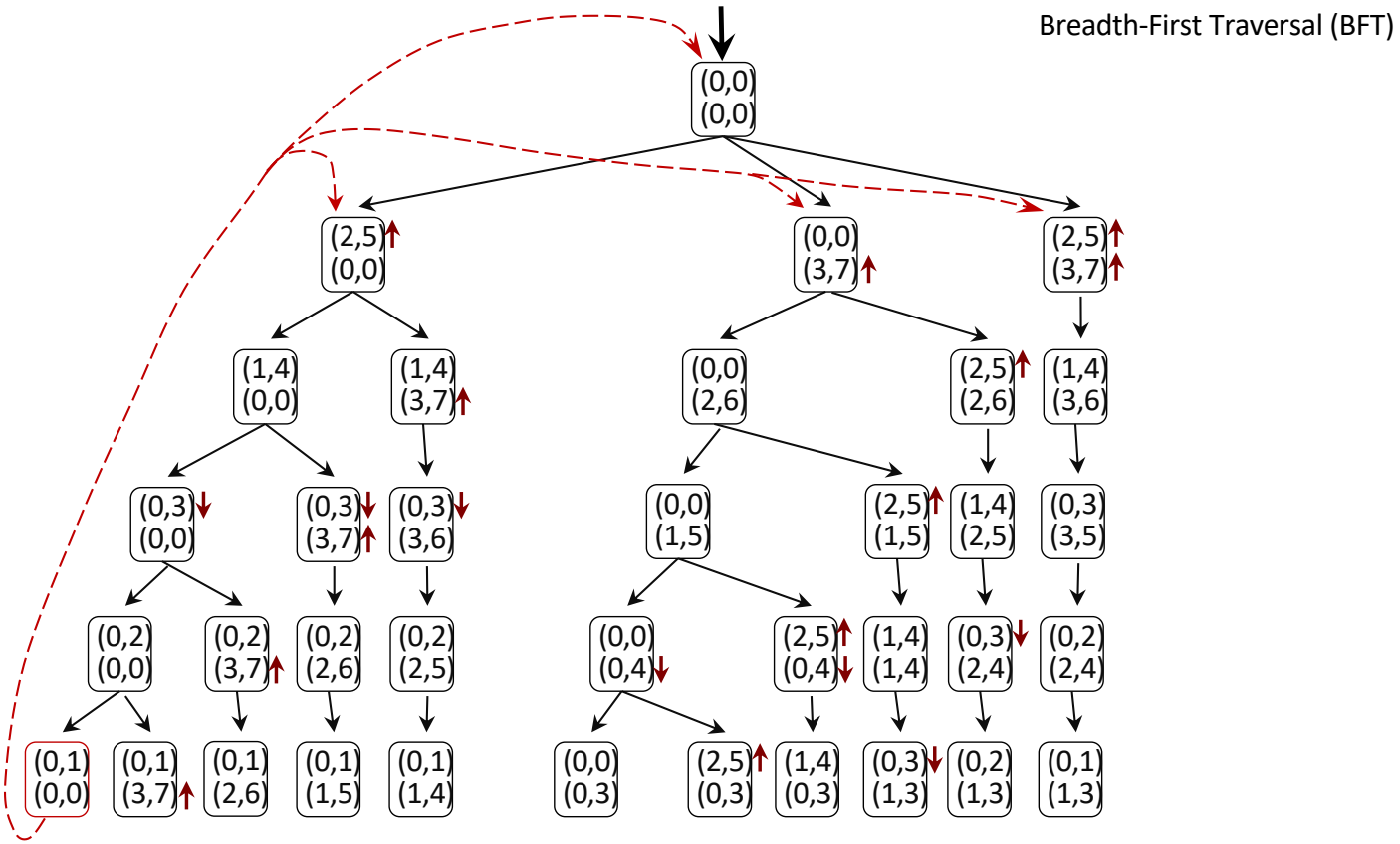

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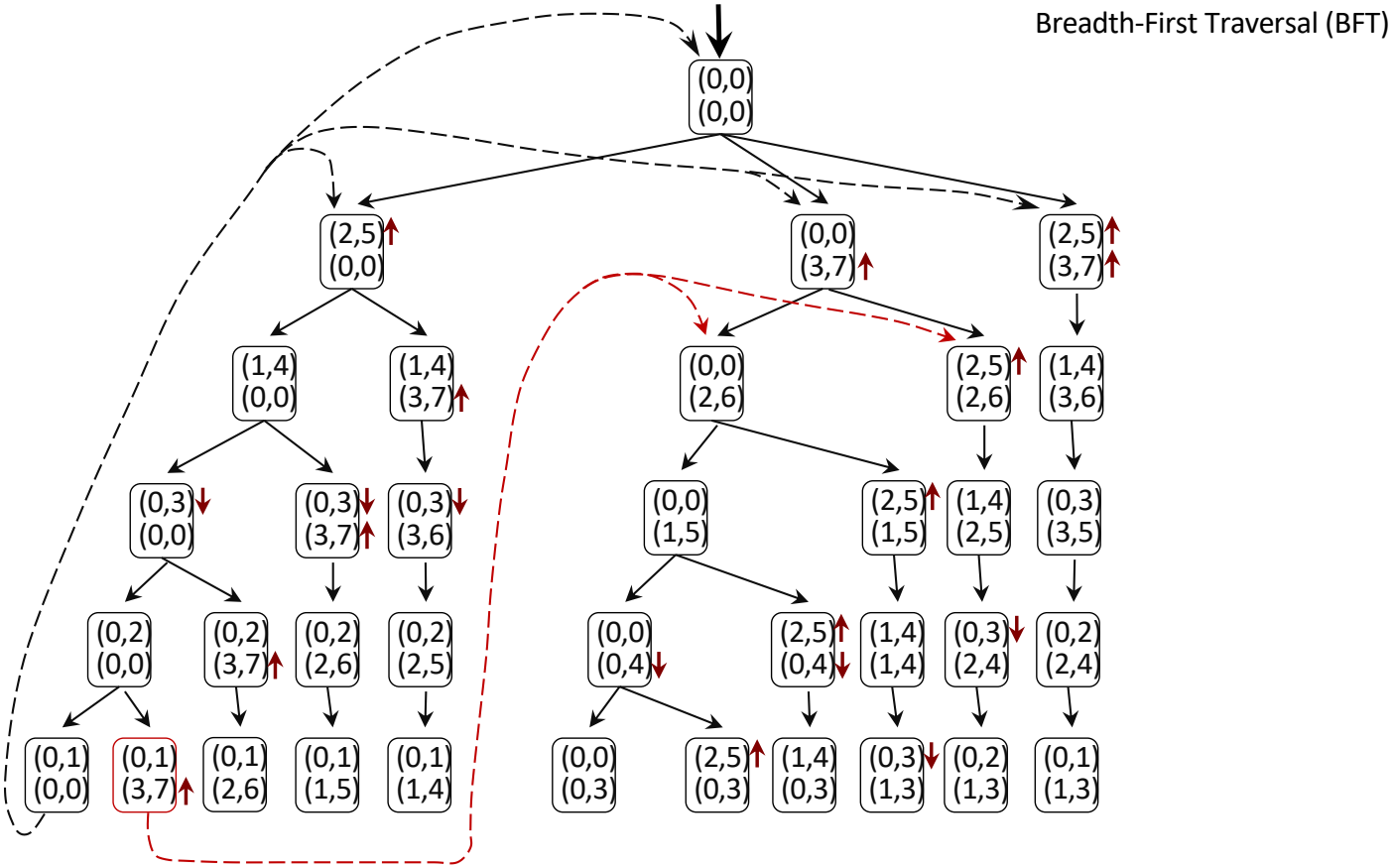

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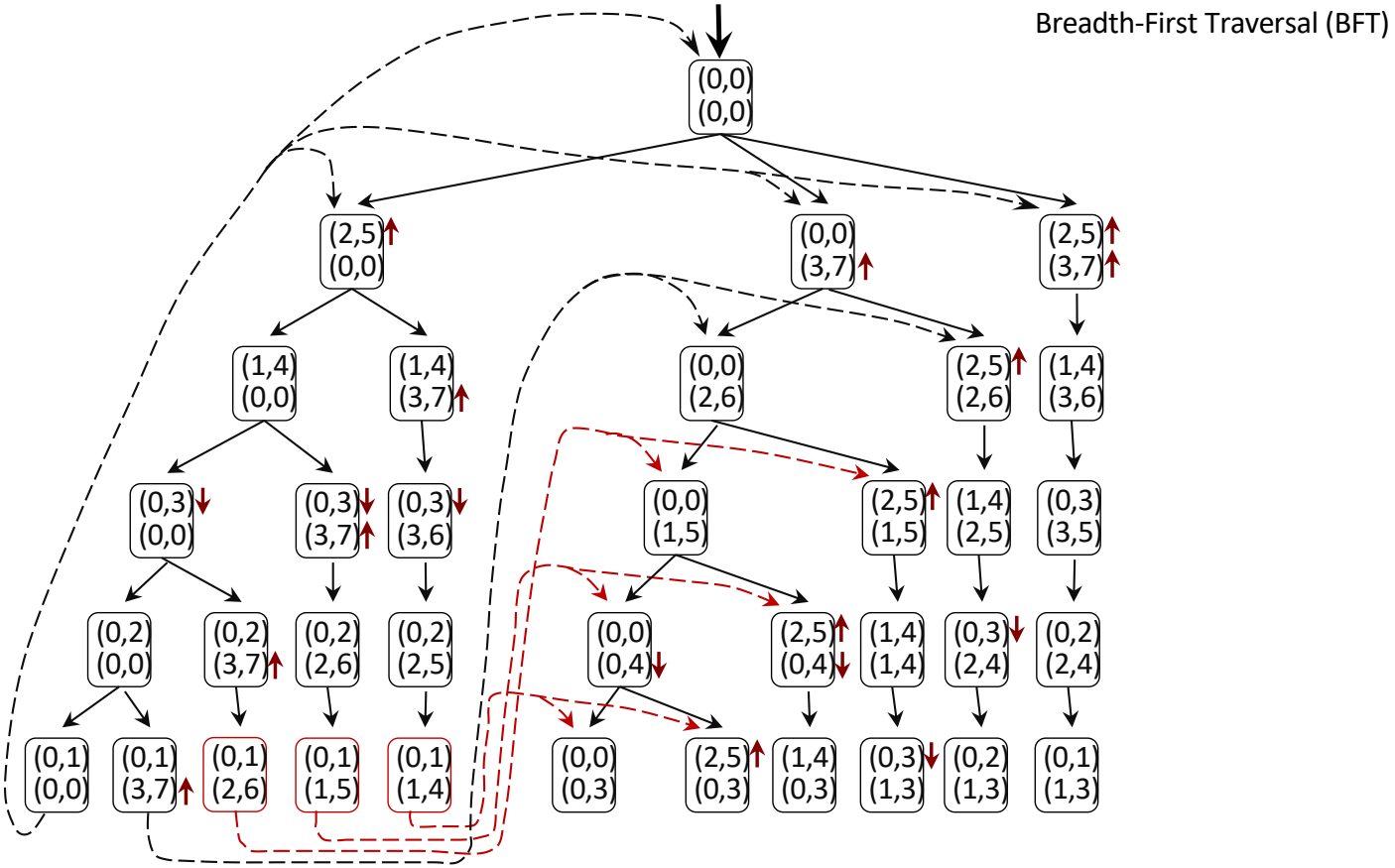

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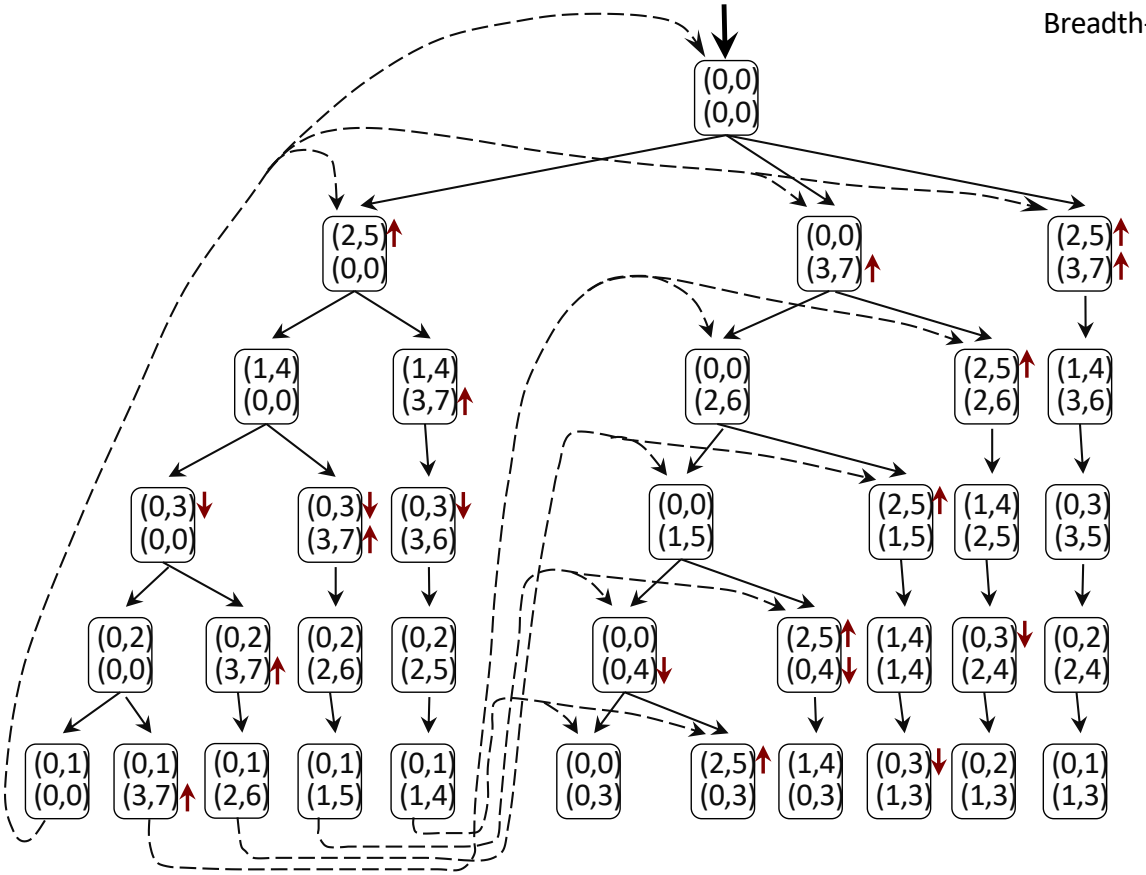

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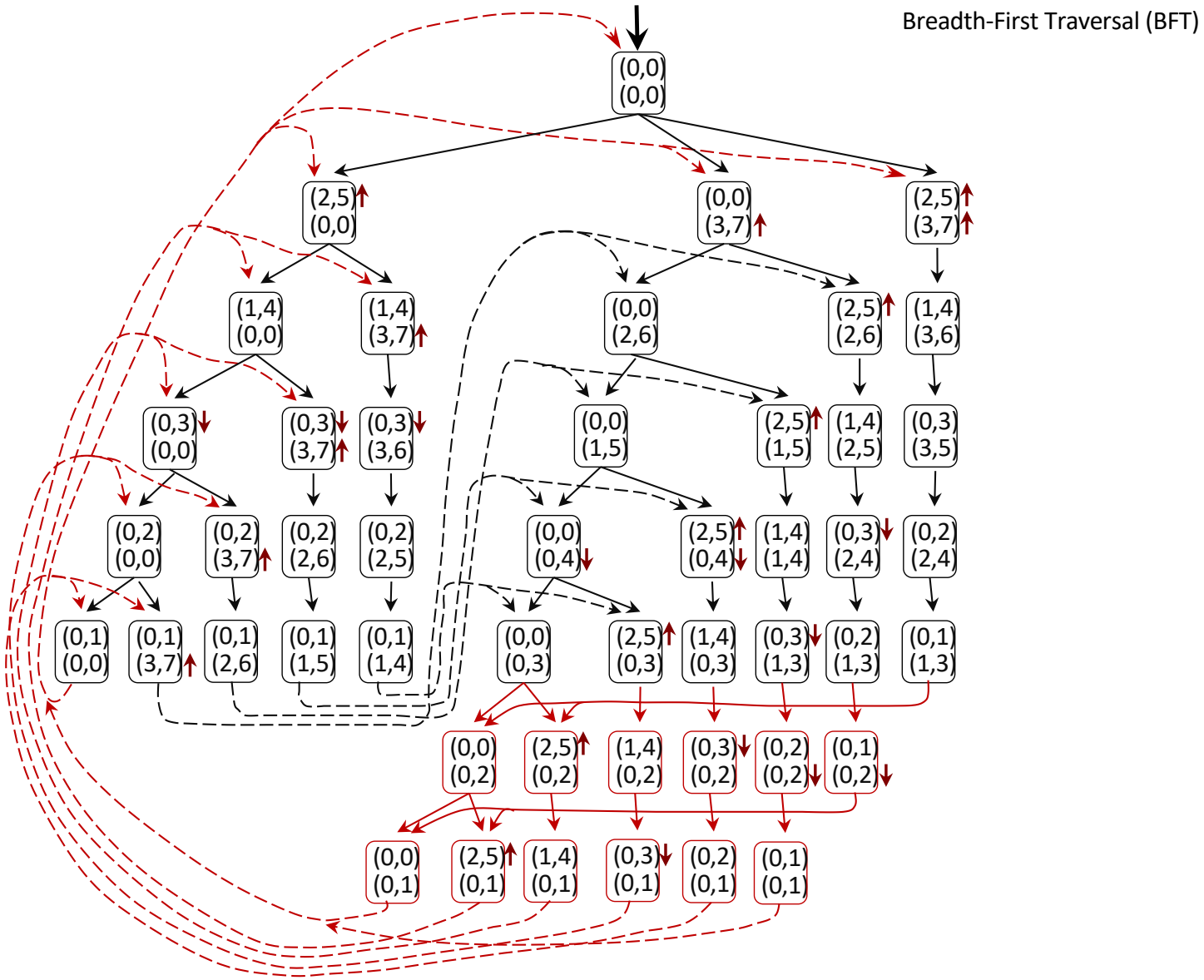

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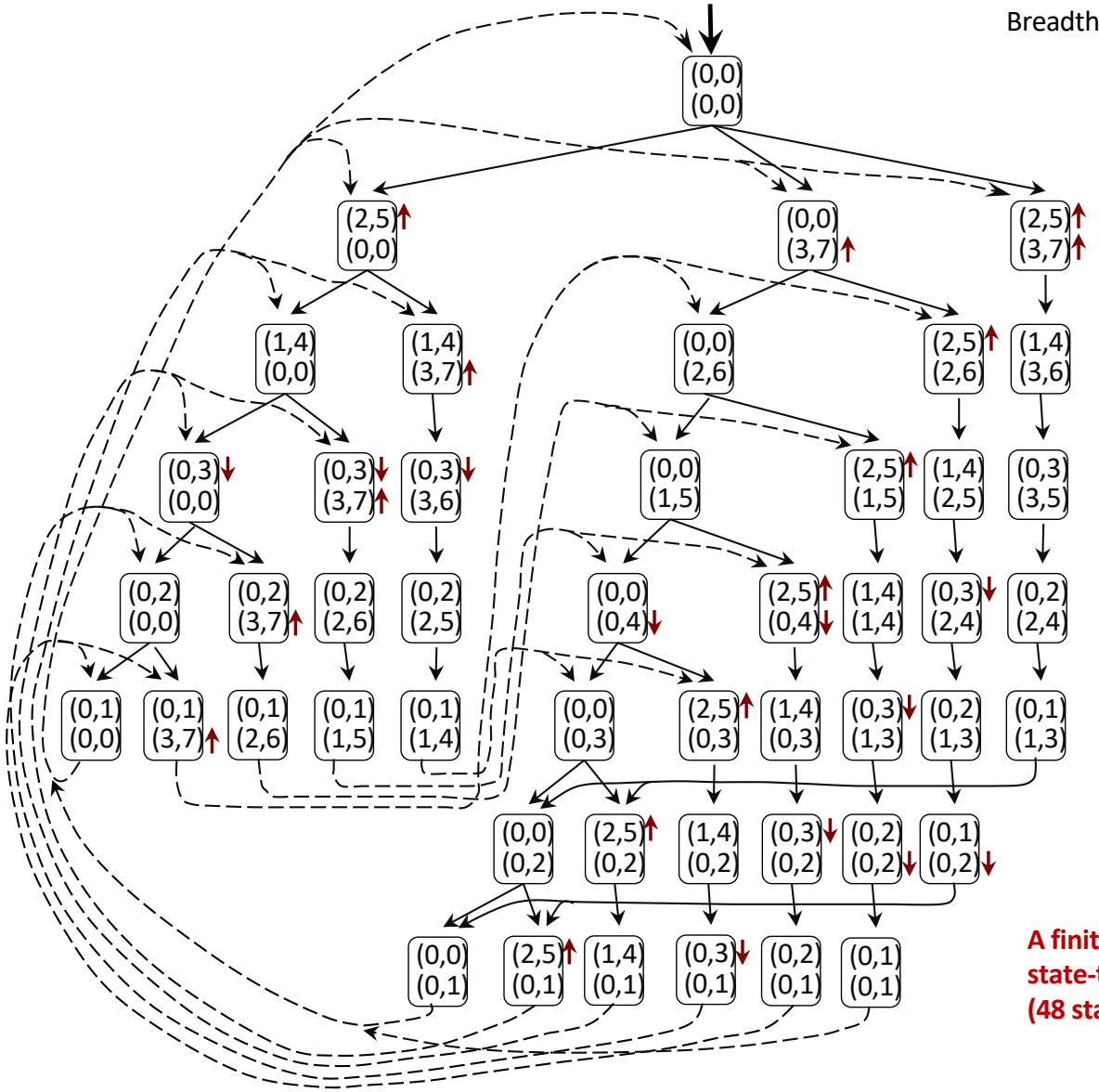
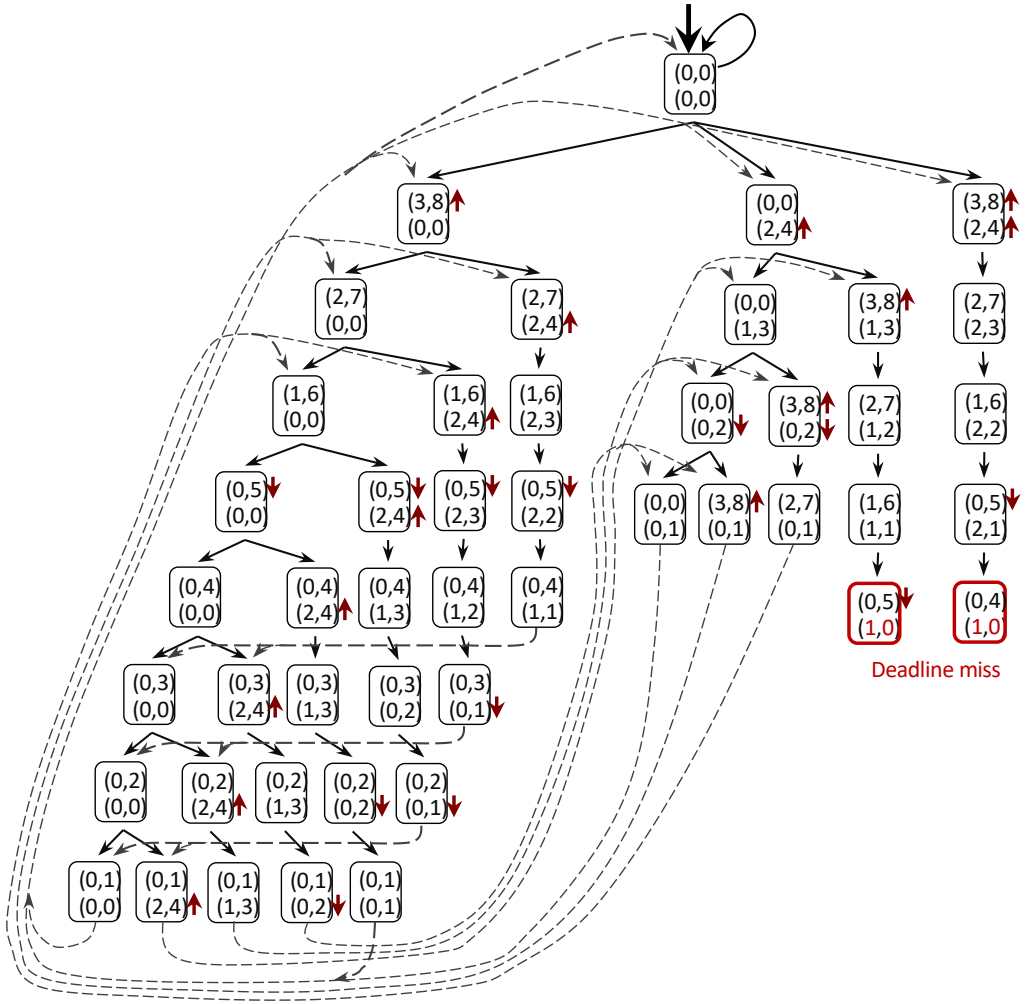


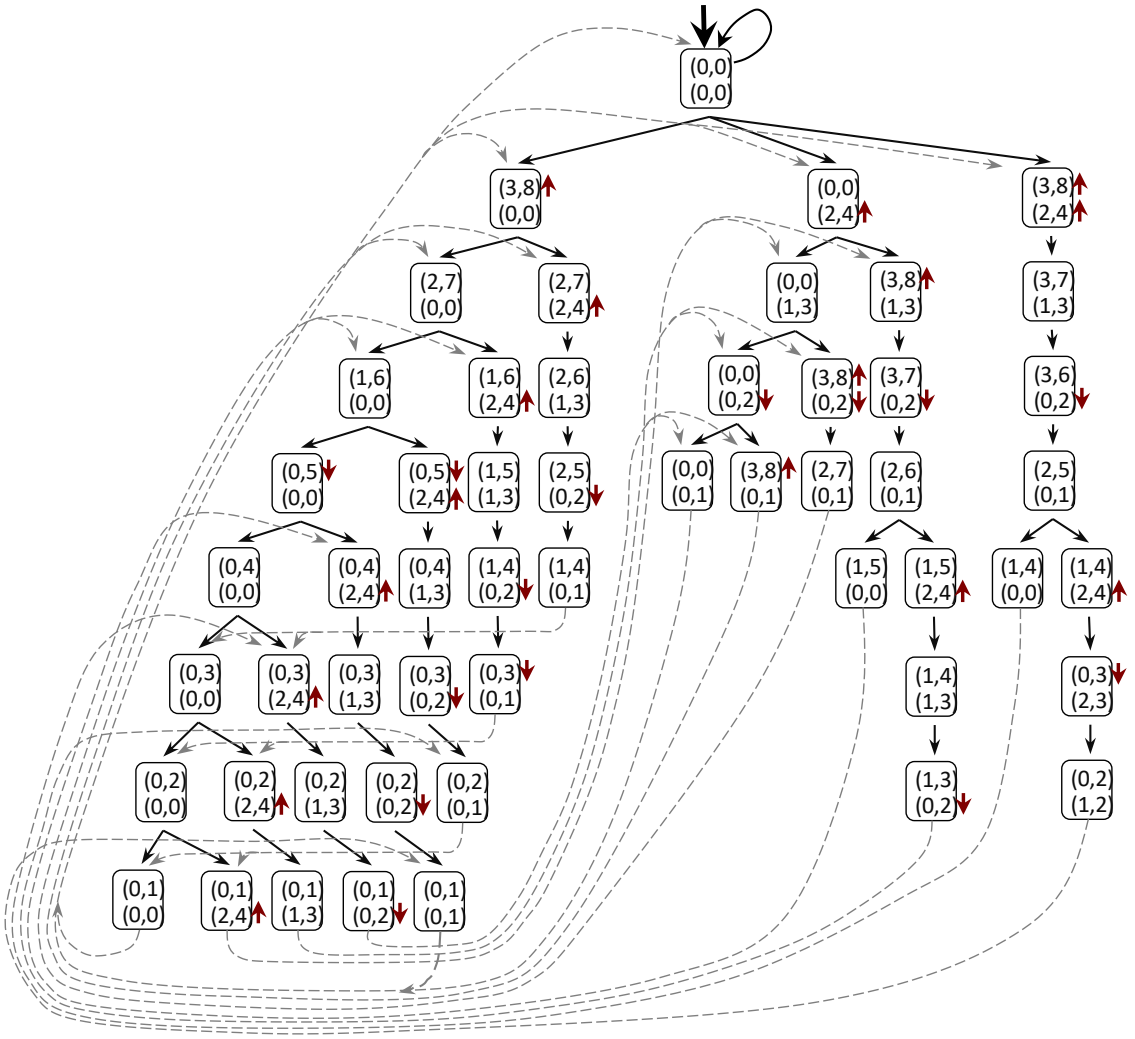
Another Example for GFP with a Deadline Miss

$\tau_1 = (3,8)$
$\tau_2 = (2,4)$
m = 1



Another Example for GEDF with NO Deadline Miss

$\tau_1 = (3,8)$
$\tau_2 = (2,4)$
m = 1



No deadline miss!
(unlike FP scheduling on the previous slide)

Another Example for 3 tasks and GEDF

$\tau_1 = (1,2)$
$\tau_2 = (2,3)$
$\tau_3 = (3,4)$
$m = 2$
$GEDF$